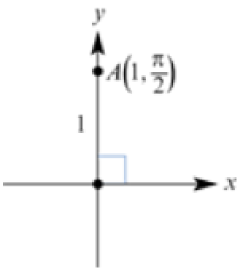
# Exercise 12H: Solutions

**1 a** We have

$$x = r\cos\theta = 1\cos\pi/2 = 0 \qquad y = r\sin\theta = 1\sin\pi/2 = 1$$

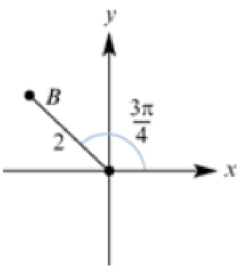
so that the cartesian coordinates are $(0, 1)$.




**b** We have

$$x = r\cos\theta = 2\cos 3\pi/4 = -\sqrt{2} \qquad y = r\sin\theta = 2\sin 3\pi/4 = \sqrt{2}$$

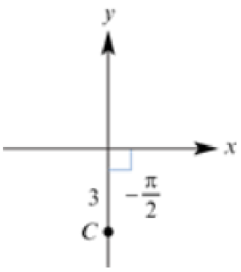
so that the cartesian coordinates are $(-\sqrt{2}, \sqrt{2})$.




**c** We have

$$x = r\cos\theta = 3\cos(-\pi/2) = 0 \qquad y = r\sin\theta = 3\sin(-\pi/2) = -3$$

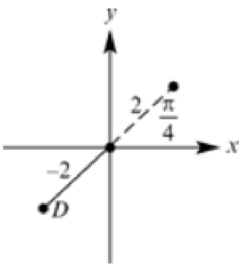
so that the cartesian coordinates are $(0, -3)$.




**d** We have

$$x = r\cos\theta = -2\cos\pi/4 = -\sqrt{2} \qquad y = r\sin\theta = -2\sin\pi/4 = -\sqrt{2}$$

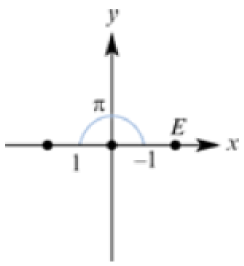
so that the cartesian coordinates are $(-\sqrt{2}, -\sqrt{2})$.



**e** We have

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$
$$= -1\cos\pi \qquad\qquad = -1\sin\pi$$
$$= 1 \qquad\qquad = 0$$

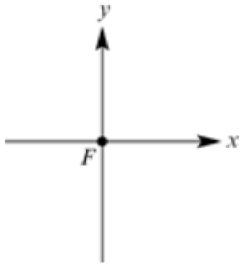
so that the cartesian coordinates are $(1, 0)$.




**f** We have

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$
$$= 0\cos\pi/4 \qquad\qquad = 0\sin\pi/4$$
$$= 0 \qquad\qquad = 0$$

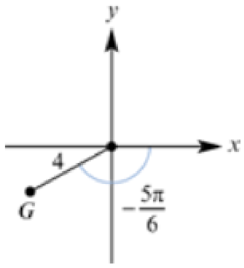
so that the cartesian coordinates are $(0, 0)$.




**g** We have

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$
$$= 4\cos -5\pi/6 \qquad\qquad = 4\sin -5\pi/6$$
$$= -2\sqrt{3} \qquad\qquad = -2$$

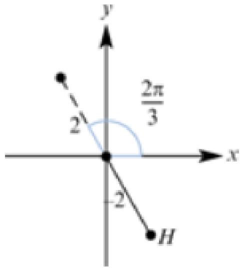
so that the cartesian coordinates are $(-2\sqrt{3}, -2)$.




**h** We have

$$x = r\cos\theta \qquad\qquad y = r\sin\theta$$
$$= -2\cos 2\pi/3 \qquad\qquad = -2\sin 2\pi/3$$
$$= 1 \qquad\qquad = -\sqrt{3}$$

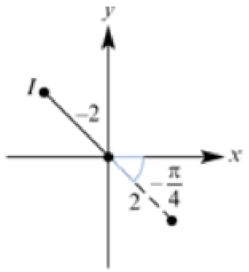
so that the cartesian coordinates are $(1, -\sqrt{3})$.



**i** We have

$$\begin{aligned} x &= r\cos\theta \\ &= -2\cos(-\pi/4) \\ &= -\sqrt{2} \end{aligned} \qquad \begin{aligned} y &= r\sin\theta \\ &= -2\sin(-\pi/4) \\ &= \sqrt{2} \end{aligned}$$

so that the cartesian coordinates are $(-\sqrt{2}, \sqrt{2})$.




**2** **a** $r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\theta = \tan^{-1} -1 = -\dfrac{\pi}{4}$

The point has polar coordinates $(\sqrt{2}, -\pi/4)$. We could also let $r = -\sqrt{2}$ and add $\pi$ to the found angle, giving coordinate $\left(-\sqrt{2}, \dfrac{3\pi}{4}\right)$.

**b** $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$

The point has polar coordinates $\left(2, \dfrac{\pi}{3}\right)$. We could also let $r = -2$ and add $\pi$ to the found angle, giving coordinate $\left(-2, \dfrac{4\pi}{3}\right)$.

**c** $r = \sqrt{(-2)^2 + 2^2} = 2\sqrt{2}$

$\theta = \tan^{-1} -1 = -\dfrac{\pi}{4}$

The point has polar coordinates $\left(2\sqrt{2}, -\dfrac{\pi}{4}\right)$. We could also let $r = -2\sqrt{2}$ and add $\pi$ to the found angle, giving $\left(-2\sqrt{2}, \dfrac{3\pi}{4}\right)$.

**d** $r = \sqrt{(-\sqrt{2})^2 + (-\sqrt{2})^2} = 2$

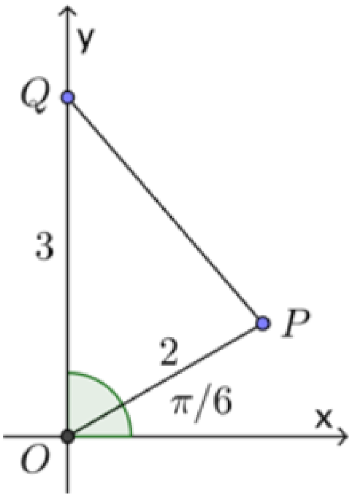
$\theta = -\dfrac{3\pi}{4}$

The point has polar coordinates $\left(2, -\dfrac{3\pi}{4}\right)$. We could also let $r = -2$ and add $\pi$ to the found angle, giving coordinate $\left(-2, \dfrac{\pi}{4}\right)$.

**e** Clearly, $r = 3$ and $\theta = 0$ so that the point has polar coordinates $(3, 0)$. We could also let $r = -3$ and add $\pi$ to the found angle, giving coordinate $(-3, \pi)$.

**f** Clearly, $r = 2$ and $\theta = -\dfrac{\pi}{2}$ so that the point has polar coordinates $\left(2, -\dfrac{\pi}{2}\right)$. We could also let $r = -2$ and add $\pi$ to the found angle, giving coordinate $\left(-2, \dfrac{\pi}{2}\right)$.

**3** Points $P$ and $Q$ are shown on the diagram below.

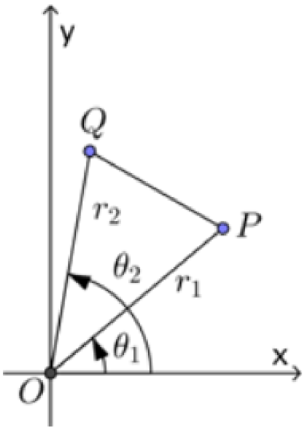



Since $\angle POQ = \frac{\pi}{3}$, we can use the cosine rule to find that

$$\begin{aligned} PQ^2 &= OP^2 + OQ^2 - 2 \cdot OP \cdot OQ \cos(\pi/3) \\ &= 2^2 + 3^2 - 2 \cdot 2 \cdot 3 \cdot \frac{1}{2} \\ &= 4 + 9 - 6 \\ &= 7. \end{aligned}$$

Therefore, $PQ = \sqrt{7}$.

**4** Points $P$ and $Q$ are shown on the diagram below.




Since $\angle POQ = \theta_2 - \theta_1$, we can use the cosine rule to find that

$$PQ^2 = {r_1}^2 + {r_2}^2 - 2r_1r_2\cos(\theta_2 - \theta_1).$$

Therefore,

$$PQ = \sqrt{{r_1}^2 + {r_2}^2 - 2r_1r_2\cos(\theta_2 - \theta_1)}.$$